
PROBLEM SET
ACM-CP SESSION 1
COMBINATORIAL GAME THEORY

Answers are available at the end of this document.
Can you solve all the problems?

1. The nim is a mathematical game of strategy where two players take turns removing objects from distinct heaps. A player must remove at least one object at his/her turn, and can remove as many as he/she wants as long as they come from a single heap. The player who removes the last object is the winner. Trump and Hillary play nim with four heaps each containing 4, 5, 6, and 7 coins. If Trump goes first, who can guarantee a win?

2. Daniel and Cody are playing a game on an infinite unit square grid. Daniel places unit square sticky notes with a letter , while Cody places unit square sticky notes with a letter . They take turns placing sticky notes on squares in the grid, with Daniel going first. Cody wants to have four of his sticky notes form the corners of a perfect square with sides parallel to the grid lines, while Daniel wants to prevent him from doing so. Can Daniel succeed? Assume that both players make optimal moves.

3. Alice and Carla are playing a game often learned in elementary school known as *Say 16*. The rules for the game are as follows: Each player takes turns saying between 1 and 3 consecutive numbers, with the first player starting with the number 1. For example, Player 1 could say the numbers 1 and 2, then Player 2 can say "3, 4, 5", then Player 1 can say "6" and so on. The goal of the game is to be the one to say “16". Carla decides that she'll go first and that Alice will go second. Is there a way to tell which player is going to win before the game even starts? Assume that each player plays "perfectly", meaning that if there was an optimal way of playing, both players would be playing the best that the game allows them to play.

4. An intelligent trader travels from 1 place to another carrying 3 sacks having 30 coconuts in each of them. No sack can hold more than 30 coconuts and he can move the coconuts between sacks. On the way, he passes through 30 checkpoints and on each checkpoint he has to give 1 coconut from each sack he is carrying. What is the maximum number of coconuts he can have after passing through every checkpoint?

5. There are 'n' pirates (Pn, Pn-1……P1) with the strict order of superiority (Pn>Pn-1>..P1). They find 100 gold coins. They must distribute the coins among themselves according to the following rules:
    1. The superior-most surviving pirate proposes a distribution.
    2. The pirates, including the proposer, then vote on whether to accept this distribution.
    3. In case of a tie vote the proposer has the casting vote.
    4. If the distribution is accepted, the coins are disbursed and the game ends. If not, the proposer is thrown overboard from the pirate ship and dies (leaving behind n-1 pirates and their original ordering), and the next most superior pirate makes a new proposal to begin the game again.

Every pirate knows that every other pirates is perfectly rational, which means:

1. They will always be able to deduce something logically deducible.
2. Their first preference is that they want to survive.
3. Their second preference is that they want to maximize the number of coins they receive.
4. Other things remaining same, they prefer to kill a pirate rather than having him alive.

What is the minimum value of 'n' such that the superior-most pirate can propose no distribution of coins that will let him survive?

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ANSWERS

1. Hillary **2**. Nope **3**. Alice Wins as long as Carla Begins **4**. 25 **5**. 203